Solved Past Papers

(From 2017, 2021)

Short Answers

(1) (2017,18)

Solve the equation $2x^2+7x+5=0$

By Factorization

$$2x^2+5x+2x+5=0.$$

$$x(2x+5)+1(2x+5)=0.$$

$$(x+1)(2x+5)=0.$$

$x+1=0$, $2x+5=0.$

$\boxed{x=-1}$, $2x=-5$

$$\boxed{x=-\frac{5}{2}}$$

(2)

Find multiplicative inverse of $\frac{7}{i-5}$

Multiplicative inverse:

$$=\frac{7}{i-5}\times\frac{i+5}{i+5}$$

$$= \frac{7(i+5)}{(i-5)(i+5)}$$

$$= \frac{7i+35}{(i)^2-(5)^2}$$

$$= \frac{7i+35}{-1-25}$$

$$= \frac{+7i+35}{-26}$$

$$= \frac{7i}{-26} + \frac{35}{-26}$$

(3) (2018, 20)

The roots of $x^2+Kx+9=0$ are equal. Find K?

$x^2+Kx+9=0 \Rightarrow a=1,\ b=K\ \&\ c=9$

Roots of given equation are equal so,

$$b^2-4ac=0.$$

$$(K)^2-4(1)(9)=0$$

$$K^2-36=0.$$

$$\sqrt{K^2}=\sqrt{36}$$

$$\boxed{K=\pm 6}$$

(4) (2018)

Simplify $\frac{3-2i}{1+5i}$

$$= \frac{3-2i}{1+5i} \times \frac{1-5i}{1-5i}$$

$$= \frac{(3-2i)(1-5i)}{(1)^2-(5i)^2}$$

$$= \frac{(3-15i-2i+10i^2)}{(1)^2-25i^2}$$

$$= \frac{3-17i+10(-1)}{1-25(-1)}$$

$$= \frac{3-17i-10}{26}$$

$$= \frac{-7-17i}{26} \Rightarrow \boxed{= -\frac{7}{26} - \frac{17i}{26}}$$

(5) (2018)

Write two consecutive integers whose sum is 41.

Required consecutive integers are x & $x+1$

$$x+(x+1)=41$$

$$2x+1=41$$

$$2x=41-1$$

$$\cancel{2}x=\cancel{40}\ 20$$

$$\boxed{x=20}$$

(6) (2019)

Define Complex numbers?

Def:- A number of the form $a+ib$ where $a,b \in R$ are real number is called complex number.

Example: $2+3i, -4+5i, 5-2i, 1+i, 1-i, -2 \pm 5\sqrt{i}$

(7) (2019)

Factorize $9a^2+16b^2$

$= (3a)^2 - (4b)^2$

$= (3a+4b)(3a-4b)$

(8) (2019)

By remainder theorem find remainder when x^2+3x+7 is divided by $x+1$

$P(x) = x^2+3x+7$

When $P(x)$ is divided by $x+1$ then remainder is:

$x = -1$

$P(-1) = (-1)^2 + 3(-1) + 7$

$= 1 - 3 + 7$

$\boxed{P(-1) = 5}$

(9) (2020 1st semester)

If $z_1 = 1+2i$ & $z_2 = 3-2i$ then find the value of $|z_1+z_2|$.

$z_1 + z_2 = (1+2i) + (3-2i)$

$= 4 + 0i$

$|z_1+z_2| = \sqrt{(4)^2+(0)^2}$

$= \sqrt{16}$

$\boxed{|z_1+z_2| = \pm 4}$

(10)

Seperate into real & imaginary part $\frac{-1+2i}{1+i}$

$$= \frac{-1+2i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{(-1+2i)(1-i)}{(1)^2-(i)^2}$$

$$= \frac{(-1+i+2i-2i^2)}{1-i^2}$$

$$= \frac{3i-1}{1+1}$$

$$= \frac{3i-1}{2}$$

$$= \frac{3i}{2} - \frac{1}{2} \Rightarrow R_e = \frac{-1}{2}, \; Im = \frac{3i}{2}$$

Long Questions

(1) (2018, 2020 1st sem)

Solve the equation $x^{1/3} - x^{1/6} - 6 = 0$

$x^{1/6} = y$ — put in eq (1)

$x^{1/3} = y^2$ — put in eq (1)

$x^{1/3} - x^{1/6} - 6 = 0$ —(i)

$y^2 - y - 6 = 0.$

$y^2 - 3y + 2y - 6 = 0.$

$y(y-3) + 2(y-3) = 0.$

$(y+2)(y-3) = 0.$

$\boxed{y=-2}$, $\boxed{y=3}$

Now,

$x^{1/6} = -2$, $x^{1/6} = 3$

$x = (-2)^6$, $x = (3)^6$

$\boxed{x = 64}$, $\boxed{x = 729}$

(2) (2018)

Find cube roots of 8.

let

$x^3 = 8$

$x^3 - 8 = 0$

$x^3 - 2^3 = 0$

$(x-2)(x^2+2x+4) = 0$

$x = 2$, $x^2+2x+4 = 0$

Use Quadratic formula

$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2}$

$= \frac{-2 \pm \sqrt{4-16}}{2}$

$= \frac{-2 \pm \sqrt{-12}}{2}$

$= \frac{-2 \pm 2\sqrt{-3}}{2} = \frac{-2 \pm 2\sqrt{3}i}{2} = 2\left(\frac{-1 \pm \sqrt{3}i}{2}\right)$

$x = 2\left(\frac{-1+\sqrt{3}i}{2}\right), 2\left(\frac{-1-\sqrt{3}i}{2}\right)$

cube root of 8 = $2, 2\omega, 2\omega^2$

(3) (2017)

$\frac{9}{x+4} + \frac{3}{x-4} = \frac{5}{x-8}$

$$\Rightarrow \frac{9(x-4)+3(x+4)}{(x+4)(x-4)} = \frac{5}{x-8}$$

$$\frac{9x-36+3x+12}{x^2-16} = \frac{5}{x-8}$$

$$(x-8)(12x-24) = 5x^2-80$$

$$12x^2-96x-24x+192 = 5x^2-80.$$

$$7x^2-120x+272=0.$$

Using Quadratic Formula:

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{120 \pm \sqrt{(120)^2-4(7)(272)}}{2(7)}$$

$$= \frac{120 \pm \sqrt{14400-7616}}{14}$$

$$= \frac{120 \pm \sqrt{6784}}{14}$$

$$= \frac{120 \pm 8\sqrt{106}}{14}$$

$$\boxed{x = \frac{60 \pm 8\sqrt{106}}{7}}$$

(4) (2019, 2021)

$$4^{1+x} + 4^{1-x} = 10$$

$$4 \cdot 4^x + 4 \cdot 4^{-x} = 10$$

$$4 \cdot 4^x + 4 \cdot \frac{1}{4^x} = 10.$$

$$2\left(2 \cdot 4^x + 2 \cdot \frac{1}{4^x}\right) = 10$$

$$2 \cdot 4^x + 2 \cdot \frac{1}{4^x} = 10/2$$

$$2 \cdot 4^x + 2 \cdot \frac{1}{4^x} = 5 \quad -(i)$$

$$4^x = y \quad - (2)$$

$$\frac{1}{4^x} = \frac{1}{y} \quad - (3)$$

Put in (1)

$$2y + \frac{2}{y} = 5$$

$$\frac{2y^2 + 2}{y} = 5 \Rightarrow 2y^2 + 2 = 5y \Rightarrow 2y^2 - 5y + 2 = 0.$$

Use quadratic formula:

$$= \frac{5 \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$

$$y = \frac{5+3}{4} = \cancel{8}^{2}/\cancel{4}, \quad y = \frac{5-3}{2} = \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \frac{1}{2}$$

$$\boxed{y = 2}, \quad \boxed{y = 1/2}$$

Put value of y in 2 & 3

$$4x = 2, \quad \frac{\cancel{1}}{\cancel{4x}} \cdot \frac{1}{2} = 4x$$

$$2^{2x} = 2^1, \quad 2^{2x} = 2^{-1}$$

$$2x = 1, \quad 2x = -1$$

$$\boxed{x = 1/2}, \quad \boxed{x = -1/2}$$

(5) (2020)

Find cube root of unity.

Let $x = (1)^{1/3}$

$x^3 = 1$

$x^3 - 1^3 = 0.$

$(x-1)(x^2 + x + 1) = 0.$

$x - 1 = 0$, $x^2 + x + 1 = 0.$

$\boxed{x = 1}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$\boxed{x = \frac{-1 \pm \sqrt{3}\,i}{2}}$$

cube root of unity:

$$\left\{1, \frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}\right\}$$

$$\omega = \frac{-1+\sqrt{3}\,i}{2}$$

$$\omega^2 = \frac{(-1+\sqrt{3}i)^2}{(2)^2} = \frac{1-\sqrt{3}i-\sqrt{3}i-3}{4} = \frac{-2-2\sqrt{3}i}{4} = \cancel{2}\left(\frac{-1-3i}{\cancel{4}\,2}\right)$$

$$= \left\{\frac{-1-\sqrt{3}\,i}{2}\right\}$$

Therefore $1, \omega, \omega^2$ are cube roots of unity.

(6) (2020 1st sem)

If α & β are roots of $5x^2 - x - 2 = 0$ form an equation whose roots are $\frac{4}{\alpha}$ & $\frac{4}{\beta}$

$$\alpha + \beta = +\frac{1}{5}$$

$$\alpha\beta = -\frac{2}{5}$$

Sum of roots:

$$\frac{4}{\alpha}+\frac{4}{\beta} = \frac{4\beta+4\alpha}{\alpha\beta} = \frac{4(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{4(1/5)}{-2/5}$$

$$= \frac{4/\cancel{8}}{-2/\cancel{8}}$$

$$= \frac{4}{-2}$$

$$= -2$$

Product:

$$\left(\frac{4}{\alpha}\right)\left(\frac{4}{\beta}\right) = \frac{16}{\alpha\beta}$$

$$= \frac{16}{-2/5}$$

$$= \frac{16}{-2}\times 5$$

$$= \frac{\cancel{80}}{-\cancel{2}}\ 40$$

$$= 40$$

So equation is

$$= x^2-2x+40$$

(7) (2021)

$$4^x - 3\cdot 2^{x+3} + 128 = 0.$$

$$2^{2x} - 3\cdot 2^x\cdot(2)^3 + 128 = 0.$$

$$2^{2x} - 24\cdot 2^x + 128 = 0.$$

$$2^x = y$$

$$2^{2x} = y^2$$

$$y^2 - 24y + 128 = 0.$$

$$y^2 - 16y - 8y + 128 = 0.$$

$$y(y-16) - 8(y-16) = 0$$

$$y - 8 = 0 \quad , \quad y - 16 = 0.$$

$$\boxed{y = 8} \quad , \quad \boxed{y = 16}$$

$$2^x = y \quad , \quad 2^x = y$$

$$2^x = 2^3 \quad , \quad 2^x = 2^4$$

$$\boxed{x = 3} \quad , \quad \boxed{x = 4}$$